

## Collaborative Problem Solving: Students Talk Their Way Into Problem-Solving Success

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### *So, what is the problem with problem solving?*

**W**e study mathematics because it helps us solve problems. Observation reveals that some students don't spend time identifying the problem, which makes it more challenging for them to create, execute, and analyze the effectiveness of a solution plan.

In his landmark 1945 book, *How to Solve It: A New Aspect of Mathematical Method*, George Pólya identified four principles of problem solving (Pólya, 1945, pp. 6–19):

1. *Understand the problem.*  
“Students are often stymied in their efforts to solve problems simply because they don't understand it fully, or even in part.”
2. *Devise a plan.*  
“The skill at choosing an appropriate strategy is best learned by solving many problems.”
3. *Carry out the plan.*  
“Using care and patience, persist with the plan you have chosen. If it continues not to work, discard it and choose another.”
4. *Look back.*  
“Take the time to reflect and look back at what you have done, what worked, and what didn't.”

In *What's Math Got to Do with It?: How Teachers and Parents Can Transform Mathematics Learning and Inspire Success*, Jo Boaler asserts that “The four stages of Pólya's cycle are neglected or missing in the work of low-achieving students, who would more typically rush into answering problems without planning systematically, neglecting to use key strategies, and finishing when they found an answer without stopping to consider whether the answer was reasonable.” (Boaler, 2008, p. 192).

Problem-solving strategies and applications relate to science, technology, and engineering as well as to everyday life. All mathematics content standards—from state-specific standards to the Common Core (<http://www.corestandards.org/Math/>)— focus on the practice and success of problem solving. They all acknowledge that *how* students learn mathematics affects *how well* they learn it.

The Common Core State Standards for Mathematical Practice (<http://www.corestandards.org/Math/Practice/>, pp. 6–8) describe in great detail how students can become mathematically proficient. Although, all mathematical practices provide students with a tool kit for problem solving, the Collaborative Problem Solving Process outlined in this paper specifically relates to the discussion and persistence that are critical shifts in a student-centered classroom. These are the proficiencies outlined in Mathematical Practice 1 and Mathematical Practice 3. Pólya’s influence on modern pedagogy (Boas, 1990) is clearly reflected in the first Mathematical Practice:

### Mathematical Practice 1: Make sense of problems and persevere in solving them.

**M**athematically proficient students [**Pólya’s step #1: Understand the problem**] *start by explaining to themselves the meaning of a problem and looking for entry points to its solution.* They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and [**Pólya’s Step #2: Devise a plan**] *plan a solution pathway rather than simply jumping into a solution attempt.* They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They [**Pólya’s Step #3: Carry out the plan**] *monitor and evaluate their progress and change course if necessary.* Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences among equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students [**Pólya’s Step #4: Look back**] *check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”* They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In addition to recommending Pólya’s four-stage plan, the first Standard for Mathematical Practice calls for students to persevere in solving problems. That requires **productive persistence**, a quality defined by the Carnegie Foundation as the “union of tenacity and good strategies.” (<http://www.carnegiefoundation.org/in-action/pathways-improvement-communities/productive-persistence/>)

*How can students build their problem-solving expertise and confidence?*

One way is by using **Collaborative Problem Solving**, a process in which teachers *facilitate* students' learning through the Standards for Mathematical Practice (<http://www.corestandards.org/Math/Practice/>, pp. 6–8) and the act of productive persistence.

Collaborative Problem Solving:

- Empowers students to reflect on their own thinking and learning.
- Enables teachers to analyze student thinking for instructional implications.
- Aligns with the Common Core Standards for Mathematical Practice and Productive Persistence.
- Can be used in K–12 classrooms.

Collaborative Problem Solving involves and engages every student in class. It also embraces the third Common Core Standard for Mathematical Practice:

### **Practice 3: Construct viable arguments and critique the reasoning of others.**

**M**athematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They *make conjectures and build a logical progression of statements to explore the truth of their conjectures*. They are able to analyze situations by breaking them into cases, and they can recognize and use counterexamples. They *justify their conclusions, communicate them to others, and respond to the arguments of others*. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to *compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is*. Elementary students can construct arguments using concrete referents, such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct even when they will not be generalized or made formal until later grades. Later, students learn to determine the domains to which an argument applies. *Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments*.

Collaborative Problem Solving can be effectively used to execute Pólya's four-stage problem-solving process (Pólya, 1945, pp. 6–19). Students use precise terms and clear statements to verbally articulate the meaning of a problem and suggest possible solution pathways. After solving and writing a draft to justify their solution strategies and reasoning, students share their responses. Then, they make revisions and complete second drafts.

**Understand:**

A problem is presented to the class. To optimize discussion, the problem should be constructed so that students can use a variety of pathways to find the solution. Students think independently about how they would paraphrase the problem. They share with a partner or in small groups. Students may be asked:

- “How would you restate the problem’s situation in your own words, preferably without using numbers?”
- “What do you need to find out? What do you know? How can you use what you know to find out what you don’t know?”
- “What other similar problems have you solved? How is this one different from those?”

**Plan:**

(a) Students think about how they would solve the problem *without actually solving it* and then verbally exchange solution *strategies* in small groups.

(b) The entire class reconvenes to discuss and compare solution strategies. Embedded in discussions are appropriate math vocabulary and sense-making justifications.

Students may be asked:

- “Which *strategy* will you choose to solve the problem? Diagramming or drawing? Working backwards? Solving a simpler problem? Explain your choice.”
- “Which *method* will you use to solve the problem? Paper and pencil? Mental math? Explain your choice.”
- “What predictions can you make about the answer? Explain your reasoning.”
- “Could you use a problem you solved before, or a simpler problem, or a more general problem, to help you devise a plan?”

**Solve:**

(a) Students solve the problem independently. Using a rubric as a guide, students write a paragraph describing their solution strategies and justifying their answers.

(b) One or two volunteers, selected by the teacher for the clarity and quality of their responses, read their first drafts to the class.

(c) Using the rubric as a guide, students rate their classmates' responses on a scale of 0–4. (The sharing student(s) typically earn a score of about 3.)

### Top-Scoring Response Rubric

#### Responses that score 4 points:

- Answer the posed question clearly and completely in a topic sentence.
- Have the correct solution.
- Use math terminology appropriately.
- Explain the solution step by step.
- Justify the solution using re-contextualizing and include a “check” or explanation that uses logical reasoning.

(d) Through a class discussion, students collaborate to upgrade the responses to full-credit anchor papers. This third discussion about the original problem solidifies conceptual understanding for the majority of students.

Students may be asked:

- “Which strategy did you execute? Why?”
- “What was the most challenging part of solving the problem? How did you face the challenge?”
- “What was the purpose of each step of your problem-solving plan?”
- “Which skills and concepts did you use to help you solve the problem?”

**Look back:**

All students reflect on the discussions and anchor papers and write a second draft. They may solve the problem in a different way and are welcome to change their solution from incorrect to correct. In their second drafts, students may also be asked to include responses to these questions:

- “If you changed your solution after the discussion surrounding the anchor paper, did you switch strategies to solve? Explain.”
- “How can you tell if your answer makes sense?”
- “What ideas and concepts did you use to (a) solve the problem and (b) assess its reasonableness?”
- “What are some things you learned by solving this problem?”

**Teacher Reflection:**

After the students’ work is done and their papers have been read, teachers should take the time to ask themselves:

- “Did my students understand the problem?”
- “What solution strategies were used? What does that reveal about my students’ conceptual understanding?”
- “What terminology did students use? What terms did they neglect to use?”
- “Did the explanatory paragraph clearly articulate the process and rationale for the solution?”

The answers to these questions inform and drive instruction for students of all ability levels through curriculum standards and instructional pedagogy.

Students use Collaborative Problem Solving to *live* the Mathematical Practices in a risk-free environment, building independence, interdependence, self-reliance, and resourcefulness as they do so.

## References

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